Mathematical Description of Disney's Principles for Data Videos

Xian Xu* HKUST
Xinyu Liu† HKUST
David Yip‡ HKUST(GZ)
Huamin Qu§ HKUST

ABSTRACT

Data videos, known as custom motion graphics, blend animated visual and auditory elements to represent a popular form of narrative visualization. However, little is known about how to manipulate existing cinematic guidelines and principles as quantifiable metrics for crafting data videos through authoring tools. To fill this gap, we draw inspiration from mathematical descriptions to visualize existing principles in measurable forms. We formulized Disney's 12 animation principles as 9 mathematical descriptions and three basic elements. Furthermore, we evaluated 12 animation principles into mathematical formulizations to match six cinematic opening styles. This confluence of mathematics and animation reveals a fresh perspective for understanding and crafting data videos for future authoring tools.

Index Terms: Mathematical Description, Animation, Data Videos.

1 INTRODUCTION

The 12 animation principles of Disney adeptly integrate time, space, and motion to judiciously present and assemble graphics, endowing characters and actions with heightened realism and vivacity. In the digital age, where data videos with custom motion graphics have emerged as a dominant form of narrative visualization, blending artful visual and auditory elements for comprehensive storytelling. Meanwhile, authoring tools for data visualization have been developed, enabling creators to craft and customize animations seamlessly. Most existing works for data visualization authoring tools mainly focus on transition techniques, automated algorithms, etc. Theoretically, researchers have presented comprehensive animated design space [6, 3, 2, 1] and studied design guidelines [4, 5] for guiding data storytellers to effectively and efficiently convey data insights.

However, little is known about how to manipulate these design guidelines in the authoring tools. The development of this thread of research still faces challenges: How can we harness the timeless wisdom of cinematic guidelines and Disney's revered principles? How do we translate them into quantifiable metrics apt for data video authoring tools? So that we can bridge the gap between traditional animation and contemporary data-driven storytelling, ensuring that our narratives resonate deeply with audiences and remain engaging in an increasingly digital world. To address these challenges, we draw inspiration from mathematics. As Socrates summarized, *"Measure and proportion manifest themselves in all areas of beauty and virtue"*. Meanwhile, mathematical expressions offer a novel perspective for understanding these cinematic guidelines and transform abstract verbal descriptions into quantifiable mathematical formulations. This transition serves as a bridge for future authoring tools and existing cinematic styles of data videos.

*e-mail: xianxu@ust.hk
†e-mail: xliuem@connect.ust.hk
‡e-mail: daveyip@ust.hk
§e-mail: huamin@cse.ust.hk

2 MATHEMATICAL DESCRIPTION FOR 12 STYLES

We use $f(x,y;t)$ to denote the animation object and it has two spatial dimensions x,y as well as one time dimension t. Due to the page limit, we present four animation principles as examples in this section.

2.1 Arcs

Arc in animation refers to the path that an object or character follows along when it moves from one point to another. It is essential to formulize the path mathematically. Here we use the parametric equations:

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

In this parametric formulation, the two spacial dimensions x,y are connected by the time parameter t. Other mathematical formulations of arcs such as implicit equations may be considered but the parametric equation description is easy to implement due to the direct relation of position and time. Other ways of expressing the arc by erasing functions or interpolation can also be fitted into this setting by some computations.

2.2 Squash and Stretch

The squash and stretch motion can be described the change of shape over time with the area preserved:

$$\text{Shape at time } t: \quad P_t(x,y) = 0$$
$$(*) \quad \text{Preserving of Certain Measure}: \quad \mathscr{M}(P_t) = \mathscr{M}(P_0)$$

The first equation is the constraint on the x,y for us to define the shape in 2D space. In $(*)$, the preserved measure $\mathscr{M}$ could be the area, perimeter of the object, or other desired measures. For example, when rendering an animation that stretches an elastic cord, it may be expected that the area of the cord object is preserved.

2.3 Solid Drawing

The principle of solid drawing means taking into account forms in three-dimensional space, or giving them volume and weight. In the mathematical formulation, we can use the geometric transformation T_i to illustrate 3D objects

$$(x,y) \to \sum_{i=1}^{n} T_i(x,y).$$

In the equation, T_i means the transformation operators that can be rotation, change of coordinate system, etc.

2.4 Timing

Timing refers to the number of drawings or frames for a given action, which translates to the speed of the action on film. We define $r_t(t)$ as the time ratio function, then the time speed of the animation is transferred as:

$$t \to r_t(t) \times t.$$

In practice, the timing principle can be illustrated by controlling the number of frames per second(fps). For example, if the preset fps = 60 and $r_t(t) = \frac{1}{2}$ for some time period. Then during the period, the fps = $60 \times \frac{1}{2} = 30$.

3 Three Basic Elements

The 12 principles described by mathematical formulas can be further abstracted into three basic elements. In this section, we first extracted these three basic elements as shape, time flow, and motion (STM). Then we use mathematics to formulate the three elements. For each element, we divided it into several different patterns based on its features or formulations. We use Shape (S) and Time Flow (T) as examples to demonstrate our mathematical formulation method.

3.1 Shape (S)

3.1.1 1D Curves

Curves are the basic 1D object in animation production. Mathematically, there are various descriptions to define the pattern of the arcs. Here we provides one example:

Parametric Arcs: In this description, we introduce a 1D parameter for us to describe the curve "dynamically". In this way, the mathematical description for the arc can have the following form:

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$

This parameter t can stand for the time and some other meaningful variable such as the rotation angle, etc. Arcs described in such form often include smooth polynomial curves or trajectories controlled by deterministic laws.

3.1.2 2D Regions

The description of 2D shapes can be various. Here we give one specific category.

Domains Enclosed by Closed Curves: The celebrated Jordan closed curve theorem says that:

Every simple closed curve in the plane separates the complement into two connected nonempty sets: an interior region and an exterior.

In this way, we can describe the 2D region by a closed curve based on Jordan closed curve theorem. In section 3.1.1, the detailed mathematical description for curves has been discussed. Readers may refer that section in order for understanding domains enclosed by closed curves.

3.2 Time Flow (T)

Unlike the real time video, the time speed in animation can be deliberately adjusted to convey emotions, set up intensity or achieve other purposes. To mathematically describe the time speed in an animation, we introduce the time ratio function defined as

$$r_t(t) = \frac{\text{animation time speed}}{\text{real time speed}} \tag{1}$$

In the equation (1), the real-time speed is a constant and $r_t = 1$ means that one second in real-time corresponds to one second of animation time, which is also known as the $1:1$ time ratio.

In the production of data videos, we can use different time ratio functions to design different animations and set up various atmosphere. For instance, designers may prefer setting $r_t > 1$ to build up intensity or $r_t < 1$ to highlight details. Various time ratio functions are provided in table for users to make different animation effects.

4 Mathematical Description for Six Attractive Openings of Data Videos

Based on [5], we provided a new perspective on the six attractive opening styles through mathematical descriptions. We first concluded the 12 principles applied in six opening styles and then used mathematical descriptions to give a general framework of these styles. We use the cinematic style Big Bag as an example:

Mathematical formulation:

$$\begin{cases} \text{Introducing object} : f(x,y,t); \\ \text{Squash and Stretch} : P_t(x,y) = 0 \text{ satisfying } \mathscr{M}(P_t) = \mathscr{M}(P_0); \\ \text{Exaggeration} : f(x,y,t) \to f(\frac{x}{s_x(t)}, \frac{y}{s_y(t)}, t); \\ \text{Solid Drawing} : (x,y) \to \sum_{i=1}^{n} T_i(x,y); \\ \text{Motion dynamics for Anticipation.} \end{cases}$$

	Big Bang
G4.1	Invoking the "Exaggeration" principle, correlate explosive imagery with the overarching theme and narrative content.
G4.2	Channel the "Anticipation" principle when frontloading content depicting chaotic scenarios.
G4.3	Merge with the "Squash and Stretch" principle when transitioning rapidly changing data content to the forefront.
G4.4	Utilize the "Arcs" principle with wide shots to panoramically display the magnitude and evolution of transformative events.
G4.5	Integrate the "Exaggeration" principle when visually presenting data with stark contrasts or exponential growth, portraying it as explosive.

Table 1: Principles for Guidelines in Big Bang

5 Discussion and Conclusion

For openings, the animation principles of "Staging", "Secondary Action", and "Appeal" can be applicable to describe a cinematic opening in "Camera Eye" as the movement of the camera directs the audience's attention and is part of the staging for appeal. The animation principles of "Secondary Action" and "Timing" can be applicable to describe a cinematic opening in "Big Bang" style. This animation principles of the non-linear interpolation of timing and spacing act as if this motion was a sudden and quick explosion expansion. This effect coincides with the "Big Bang" effect as one of the six cinematic opening styles.

Data video creation stands as an emerging field that marries data visualization with the nuances of cinematic arts. Rooted in visual and auditory facets, it heralds a fresh perspective in representing information. By mathematically translating Disney's Animation Principles, we can illuminate the art of cinematic openings and endings in data videos. Further exploration in this confluence will inspire the community to develop richer narratives, broader applications, and more engaging user experiences.

References

[1] B. Bach, Z. Wang, M. Farinella, D. Murray-Rust, and N. Henry Riche. Design patterns for data comics. In *Proceedings of the ACM Conference on Human Factors in Computing Systems*, pp. 1–12. ACM, Montréal, QC, Canada, 2018. 1

[2] X. Lan, Y. Shi, Y. Wu, X. Jiao, and N. Cao. Kineticharts: Augmenting affective expressiveness of charts in data stories with animation design. *IEEE Transactions on Visualization and Computer Graphics*, 28(1):933—943, 2021. 1

[3] Y. Shi, X. Lan, J. Li, Z. Li, and N. Cao. Communicating with motion: A design space for animated visual narratives in data videos. In *Proceedings of the ACM Conference on Human Factors in Computing Systems*, pp. 1–13, 2021. 1

[4] X. Xu, A. Wu, L. Yang, Z. Wei, R. Huang, D. Yip, and H. Qu. Is it the end? guidelines for cinematic endings in data videos. In *Proceedings of the 2023 chi conference on human factors in computing systems*, pp. 1–20. ACM CHI, Hamburg, Germany, 2023. 1

[5] X. Xu, L. Yang, D. Yip, M. Fan, Z. Wei, and H. Qu. From 'wow' to 'why': Guidelines for creating the opening of a data video with cinematic styles. In *Proceedings of the 2022 CHI Conference on Human Factors in Computing Systems*, CHI '22. Association for Computing Machinery, New York, NY, USA, 2022. doi: 10.1145/3491102.3501896 1, 2

[6] L. Yang, X. Xu, X. Lan, Z. Liu, S. Guo, Y. Shi, H. Qu, and N. Cao. A design space for applying the freytag's pyramid structure to data stories. *IEEE Transactions on Visualization and Computer Graphics*, 28:922–932, 2021. doi: 10.1109/TVCG.2021.3114774 1